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Book Reviews

Anna Nagurney and Ding Zhang, *Projected Dynamical Systems and Variational Inequalities with Applications*, International Series in Operations Research and Management Science, Vol. 2, Kluwer Academic Publishers, Dordrecht / Boston / London, 1996, 320 pages, ISBN 0-7923-9637-5, US$ 110.00.

The problem of equilibrium of competitive systems arising in different disciplines is very important to the understanding of the behaviour of these systems. The complexity and the large-scale character of such systems have led to the development of a variety of mathematical tools for their analysis, the discussion of their qualitative behaviour, and the corresponding computation.

One of these methodologies is the theory of variational inequalities, which today is a powerful tool for both the qualitative analysis of equilibria arising in entirely distinct equilibrium problems, as well as, the formulation of rigorous computational procedures. Indeed algorithms for the computation of variational inequality problems have performed well in practice and are suitable for many large-scale applications. Moreover, one should note that the variational inequality problem contains in the case of finite dimensions as special cases several well-known problems in mathematical programming, e.g. the complementarity problems, certain optimization problems, certain multivalued equations etc. This is beneficial from the point of view of unification and also from the point of view of the offered comparison possibilities. Several important equilibrium problem can be treated by means of variational inequalities: determination of profit, maximizing production outputs of competing firms, computation of the optimal commodity production, consumption, and interregional trade patterns in spatial economic systems, determination of cost-minimizing routes of travel etc.

The book applies the theory of variational inequalities to the solution of dynamic problem. Note that the classical dynamical system theory permits the study of the dynamic behaviour of many systems, but it cannot adequately model problems where financial, natural, or human resources, give rise to differential equations with discontinuous right hand side.

In order to achieve the study of such system the author presents and develops in this book a new type of dynamical system, which is called a projected dynamical system. This type allows the modeler to incorporate constraints and to ensure that, e.g. prices, production outputs, etc. are nonnegative over time. The method of projected dynamical systems leads to ordinary differential equations, which have

stationary points that coincide with solutions of associated variational inequality problems. Thus any equilibrium problem which leads to a finite-dimensional variational inequality problem leads also to a projected dynamical system. The scope of the present book is the in- depth study of the projected dynamical systems. The main difference from the classical dynamic systems is that the right-hand side of the ordinary differential equation is defined through a projection operator, and thus it is no longer continuous. Obviously the discontinuities result from the constraints of the problem in question.

The book is divided into two parts: Part I gives the theory of projected dynamical systems and variational inequalities and part II applies the theory to a variety of appropriately selected competitive equilibrium problems taken from operations research, management science, and economics.

Part I consists of four chapters. Chapter 1 is an introduction and overview of the material of the book. Chapter 2 deals with the variational inequality problem and introduces the notion of projected dynamical systems. Then their relation to the theory of variational inequalities is studied. Then the expected behaviour of the solutions is discussed and important issues such as the feasibility, the discrete-time versions of the differential equations, as well as the existence and uniqueness of the solution path are also addressed.

In Chapter 3 the theoretical study continues with the stability analysis. Both the "regularity approach" and the "monotonicity approach" are used for the stability analysis. The dynamical stability analysis of this Chapter permits the distinction between those variational inequality problems having "good" stability of equilibria and those with "bad" stability.

Chapter 4 deals with the computation of the stationary points of projected dynamical systems. The author presents a general iterative scheme that contains, as special cases, many of the numerical methods of the field of dynamical systems. Moreover a proof of convergence is given and the conditions for convergence are interpreted in the context of specific applications.

Part II begins with Chapter 5, dealing with oligopolistic market equilibrium problems, which are examples of game theory problems. Two models are considered in this Chapter of the book: the classical, Cournot-Nash equilibrium model, and then the spatial, or network oligopoly model, in which firms and demand markets may be spatially separated. Both models are examples of imperfectly competitive models. First the variational inequality formulations are derived and then the projected dynamical systems are formulated and their interpretations are given. Then the stability analysis of the two models, using the two distinct approaches developed in Chapter 3 is performed. Finally, an algorithm is proposed for the computation of the solutions, and convergence results, are given. Numerical results illustrate the whole procedure.

In Chapter 6 a spatial price equilibrium problem is studied. This problem has wide applications from the commodity and energy markets to international trade and is an example of perfect competition. First the variational inequality formu-

lation is given. Then the authors obtain the projected dynamical systems model and give an interpretation of it. Also stability analysis is conducted by applying and extending the results of Chapter 3, and an algorithm is proposed, together with the corresponding convergence results. Finally, numerical examples illustrate the theory. The authors have implemented the numerical schemes on (massively) parallel architectures. This last fact is important for both the book and the reader because spatial price equilibrium problems can be large-scale in real problems.

Chapter 7 and Chapter 8 are devoted to the study of the traffic network equilibrium problem. Starting from the variational inequality formulations known as the Wardropian equilibrium conditions, the authors derive projected dynamical systems models in the case of elastic demands and then fixed demands in Chapter 7 and 8 respectively. Stability results are also obtained by means of the theory of Chapter 3. The authors present algorithms based on the general iterative scheme both for the elastic demand models of Chapter 7 and for the fixed demand model of Chapter 8. Again convergence results are given, as well as numerical examples. All the applications are carefully selected by the authors. They are problems of general interest which parallel, illustrate different equilibrium concepts. Moreover they highlight in the best possible way the theory presented here. Enough figures are included in the text helping the reader to understand the basic concepts. Sources and notes follow each chapter. References are also given after each chapter for the reader's convenience. Another strong point of the book is that each application chapter in the second part of it is selfcontained. The present book is addressed to an audience of students, researchers and practitioners in management science, operations research, and economics, who are interested in the formulation, analysis, and computation of equilibrium problems in a dynamical setting.

The results of the book are original, very interesting and significantly adding to the related published works. The book is properly structured, is written with accuracy and clarity, and has appropriate referencing. It should therefore be considered an essential piece of work for everyone in the relevant fields to read.

P.D. Panagiotopoulos
Institute of Steel Structures,
Aristotle University,
Thessaloniki, Greece

Donald H. Hyers, George Isaac and Themistocles M. Rassias, *Topics in Nonlinear Analysis and Applications*. World Scientific Publishing Co., Singapore–New Jersey–London–Hong Kong, 1997, XIII+699 pp.

The book explores several new interconnections and interrelations between mathematical analysis and topology and their applications. Main emphasis is given to mathematical research that has been carried through, at an international level, mainly during the last years. It presents an impressive body of research in nonlinear